Recall

Vector superfield $V = V^{\dagger}$ (gauge bosons + superpartners)

$$V(x,\theta,\bar\theta) = C + i\theta\chi - i\bar\theta\bar\chi + \frac{1}{2}i\theta\theta(M+iN) - \frac{1}{2}i\bar\theta\bar\theta(M-iN)$$
$$+\theta\sigma^{\mu}\bar\theta V_{\mu} + \theta\theta\bar\theta(\bar\lambda + \frac{1}{2}\bar\sigma^{\mu}\partial_{\mu}\chi)$$
$$+\bar\theta\bar\theta\theta(\lambda - \frac{1}{2}\sigma^{\mu}\partial_{\mu}\bar\chi) - \frac{1}{2}\theta\theta\bar\theta\bar\theta(D + \frac{1}{2}\partial_{\mu}\partial^{\mu}C)$$

$C, M, N, D, V_{\mu}$ real

$\underbrace{\lambda, \bar\lambda, V^{\mu\nu}, D}_{\text{transform among themselves}}$, $\quad V^{\mu\nu} = \partial^{\mu}V^{\nu} - \partial^{\nu}V^{\mu}$

$= U(1)$ field strength

Supersymmetric $U(1)$ gauge transformation:

$$V'(x,\theta,\bar\theta) = V(x,\theta,\bar\theta) + i(\Lambda - \Lambda^{\dagger})$$

$\Lambda$ = chiral superfield

$\lambda, \bar\lambda, V^{\mu\nu}, D$ invariant in gauge transformations

Wess-Zumino gauge:

$$V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^{\mu}\bar\theta V_{\mu} + \theta\theta\bar\theta\bar\lambda + \bar\theta\bar\theta\theta\lambda - \frac{1}{2}\theta\theta\bar\theta\bar\theta D$$

Field strength superfield:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$$

Gauge field kinetic terms from

$$\frac{1}{2}(W_\alpha W^\alpha)_F = -\frac{1}{4}V^{\mu\nu}V_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda}$$

$$-\frac{1}{4}V^{\mu\nu}(*V_{\mu\nu}) + \frac{1}{2}D^2$$

To describe fermions, need two chiral superfields $\psi_D = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix} = \begin{pmatrix}\psi_\alpha\\ \bar{\chi}^{\dot\alpha}\end{pmatrix}$; call them $S, T$; charges of $S, T$ opposite

$\Rightarrow$ gauge field - fermion interactions, fermion mass terms

$$\mathcal{L} = \frac{1}{2}(W_\alpha W^\alpha)_F + (S^\dagger e^{2qV}S + T^\dagger e^{-2qV}T)_D$$

$$+ m(ST + S^\dagger T^\dagger)_F$$

Scalar, fermion interactions from superpotential $W$:

$\mathcal{L}_W = W_F + h.c.$, includes scalar potential. F-term part (D-term from $W_\alpha W^\alpha$) Scalar potential form determines the possible spontaneous breaking of gauge symmetry. and susy.

## Breaking supersymmetry: vector superfield

Previously the spontaneous breaking of supersymmetry via F-field was discussed. In the case of a vector superfield, the susy transformations of the fields (p.53) indicate that $\lambda_\alpha$ transformation can get a vev without breaking Lorentz invariance:

$$\langle \delta\lambda_\alpha \rangle = i\xi_\alpha \langle D \rangle$$

Indeed, since $V = |F|^2 + \frac{1}{2}|D|^2$, $V > 0$ for $\langle D \rangle \neq 0$ and no supersymmetric minima occur.

An example of D-term breaking is the Fayet-Iliopoulos (1974) mechanism with $U(1)$ gauge invariance. This utilizes the fact that the D-term is gauge invariant and gives a supersymmetric action. Thus a term proportional to D can be added to the Lagrangian:

$$\mathcal{L}_D = \xi D(x)$$

This term changes the eq. of motion for D (p.58) to

$$D = -(\xi + \sum e_i \varphi_i^\dagger \varphi_i)$$

(Note that without $\xi$-term, $\langle D \rangle = 0$, if $\varphi_i = 0$ values are allowed, and susy would not break.)

Assuming that there is no superpotential and one chiral superfield $\Phi$, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(W^\alpha W_\alpha)_F + (\Phi^\dagger e^{2eV}\Phi)_D + \xi D$$

with $D = -(\xi + e\varphi^\dagger\varphi)$

If $\xi e<0$, in the minimum $\langle\varphi\rangle^2=-\frac{\xi}{e}$ and $V=0$, i.e. supersymmetry is preserved, but gauge symmetry is broken.

On the other hand, if $\xi e>0$, the minimum is achieved with $\langle\varphi\rangle=0$, i.e. gauge symmetry is preserved, but $V(\langle\varphi\rangle)=\frac{1}{2}\xi^2>0$ and supersymmetry is broken.

$$V(\varphi)=\frac{1}{2}(\xi+e\varphi^{+}\varphi)^2$$

implies that $m_\varphi^2=e\xi$, while no mass terms are generated for $\psi$. Since gauge symmetry is unbroken, $V_\mu$ remains massless. $\chi$-field remains massless as well - it is the goldstino of the model.

In the model on p. 58 a similar effect on the scalar sector is seen:

$$\frac{1}{2}q^2(\varphi_S^{+}\varphi_S-\varphi_T^{+}\varphi_T)^2 \rightarrow \frac{1}{2}[\xi+q(\varphi_S^{+}\varphi_S-\varphi_T^{+}\varphi_T)]^2$$

shifts the masses

$$m_S^2 \rightarrow m^2+q\xi$$
$$m_T^2 \rightarrow m^2-q\xi$$

while the fermion mass remains $m_\psi=m$.

— x —

In the absence of gravitational corrections, the tree level mass formula is (c.f. p46)

$$\sum_J(-1)^{2J}(2J+1)\,m_J^2 = 2\sum_a\langle D^a\rangle\,\mathrm{Tr}\,T^a$$

For non-abelian generators $\mathrm{Tr}\,T^a=0$.
In SM $\mathrm{Tr}\,Y=0$, as well as in minimal susy extension

# Nonabelian gauge theory with supersymmetry

In Nature one expects that supersymmetry is realized at energies where the Standard Model gauge group $SU(3)\times SU(2)\times U(1)$ is unbroken. Thus one needs to generalize the treatment of $U(1)$ gauge symmetry to nonabelian case.

Assume that Hermitian matrices $t^a$ form a representation of the internal symmetry group $G$, $[t^a, t^b] = i f^{abc} t^c$. The chiral field $\phi$ in this representation transforms as

structure constants

$$\phi \to \phi' = e^{-2igt^a\Lambda^a}\phi,$$

$$\phi^\dagger \to \phi^{\dagger\prime} = \phi^\dagger e^{2igt^a\Lambda^{a\dagger}}$$

where $\Lambda^a$ are chiral superfields.

If $V$ ($V = 2gV^a t^a$) transforms as $e^V \to e^{V'} = e^{-i\Lambda^\dagger} e^V e^{i\Lambda}$ ($\Lambda = 2g\Lambda^a t^a$)

then

$$\phi^\dagger e^{2gt^aV^a}\phi$$

is gauge invariant. So, one needs to find a suitable $\Lambda$:

$$e^{V'} - e^V = e^{V+\delta V} - e^V$$

$$= e^{-i\Lambda^\dagger} e^V e^{i\Lambda} - e^V$$

Hausdorff: $e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[A,[A,B]]-\frac{1}{12}[B,[B,A]]}$

$$= e^{-i\Lambda^\dagger} e^{V+i\Lambda+\frac{i}{2}[V,\Lambda]+\frac{i}{12}[V,[V,\Lambda]]+\dots} - e^V$$

$$= e^{V+i(\Lambda-\Lambda^\dagger)+\frac{i}{2}[V,(\Lambda+\Lambda^\dagger)]+\frac{i}{12}[V,[V,(\Lambda+\Lambda^\dagger)]]+\dots} - e^V$$

Thus $\delta V = i(\Lambda-\Lambda^\dagger) + \frac{i}{2}[V, \Lambda+\Lambda^\dagger] + \frac{i}{12}[V,[V,\Lambda+\Lambda^\dagger]] + \dots$

which is in terms of the superfields

$$(V^{a'} - V^a)\, t^a$$

$$= \left\{ i(\Lambda^a - \Lambda^{a+}) t^a + ig V^b (\Lambda^c + \Lambda^{+c}) \underbrace{[t^b, t^c]}_{if^{bca} t^a} + \frac{i}{3} g^2 V^b V^c (\Lambda^d + \Lambda^{+d}) [t^b, \underbrace{[t^c, t^d]}_{if^{cde} t^e}] + \dots \right\}$$

$$= \{ \underbrace{i(\Lambda^a - \Lambda^{a+}) \quad - g f^{bca} V^b (\Lambda^c + \Lambda^{+c})} - \frac{i}{3} g^2 f^{cde} f^{bea} V^b V^c (\Lambda^d + \Lambda^{+d}) \} \, t^a$$

$\theta\sigma\bar\theta$ part: $\quad \partial_\mu(\varphi^a + \varphi^{a+}) + g f^{cba} V^b_\mu (\varphi^c + \varphi^{c+})$

So the first two terms generate the usual gauge transformations. Generally, if $V'^a$ is also wanted in WZ gauge ($V^{a'} V^{b'} V^{c'} = 0$), one has to choose $\Lambda_a$ such that

$$V^a V^b \Lambda^c = 0$$

in which case

$$\delta V^{WZ} = i(\Lambda - \Lambda^+) + \frac{i}{2} [V^{WZ}, \Lambda + \Lambda^+]$$

In order to generalize the field strength superfield to the nonabelian case, one needs to have gauge and supersymmetric covariant derivatives, $\nabla_A$, $A = \mu, \alpha, \dot\alpha$.

Then

$$(\nabla_A \Phi)' = e^{-i\Lambda} \nabla_A \Phi; \qquad \Phi \to \Phi' = e^{-i\Lambda} \Phi$$

| $\bar\Phi D_\mu \Phi$ gauge invariant

i.e. $\quad \nabla'_A = e^{-i\Lambda} \nabla_A e^{i\Lambda}$

For a chiral superfield

$$\bar D_{\dot\alpha} \Lambda = 0 = D_\alpha \Lambda^+$$

Let's choose $\nabla_{\dot\alpha} = \bar D_{\dot\alpha}$ and thus $\nabla'_{\dot\alpha} = \nabla_{\dot\alpha}$

For $\nabla_\alpha$ one can take

$$\nabla_\alpha = e^{-V} D_\alpha e^{V}:$$

$$\nabla'_\alpha = e^{-V'} D_\alpha e^{V'} = e^{i\Lambda} e^{-V} e^{i\Lambda^+} \underbrace{D_\alpha e^{-i\Lambda^+}}_{e^{-i\Lambda^+} D_\alpha} e^{V} e^{i\Lambda}$$

$$= e^{-i\Lambda} \nabla_\alpha e^{i\Lambda}$$

Abelian field strength superfield: $W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V$

In the non-abelian case try $W_\alpha = \frac{1}{2g} \bar{D}^2 e^{-V} (D_\alpha e^{V})$

This transforms as

$$2g\, W'_\alpha = \bar{D}^2 e^{-V'} (D_\alpha e^{V'})$$

$$= \underbrace{\bar{D}^2 e^{-i\Lambda}}_{e^{-i\Lambda} \bar{D}^2} e^{-V} e^{i\Lambda^+} \underbrace{D_\alpha e^{-i\Lambda^+}}_{e^{-i\Lambda^+} D_\alpha} e^{V} e^{i\Lambda}$$

$D_\alpha \Lambda^+ = 0$, $\bar{D}_{\dot\alpha} \Lambda = 0$

$$= e^{-i\Lambda} \bar{D}^2 e^{-V} (D_\alpha e^{V}) e^{i\Lambda} + e^{-i\Lambda} \underbrace{\bar{D}^2 D_\alpha e^{i\Lambda}}_{\underbrace{\bar{D}_{\dot\beta} \epsilon^{\dot\beta\dot\gamma} \underbrace{\{\bar{D}_{\dot\gamma}, D_\alpha\}}_{2i\sigma^\mu{}_{\alpha\dot\gamma}\partial_\mu} e^{i\Lambda}}_{= \epsilon^{\dot\beta\dot\gamma} 2i\sigma^\mu{}_{\alpha\dot\gamma} \partial_\mu \underbrace{\bar{D}_{\dot\beta} e^{i\Lambda}}_{=0}}}$$

$$= 2g\, e^{-i\Lambda} W_\alpha e^{i\Lambda}$$

Expanding $e^{V_{wz}} = 1 + V_{wz} + \frac{1}{2} V^2_{wz}$

Then $(e^{-V} D_\alpha e^V)_{wz} = (1 - V + \frac{1}{2}V^2)(D_\alpha (1 + V + \frac{1}{2}V^2))$

$$= D_\alpha V + \frac{1}{2}(D_\alpha V)V + \frac{1}{2}V(D_\alpha V) - V D_\alpha V$$

$$= \left(D_\alpha V + \frac{1}{2}[D_\alpha V, V]\right)_{wz}$$

Thus $W_\alpha = W^a_\alpha t^a$

$$= \frac{1}{2g} \bar{D}^2 \left\{ D_\alpha 2g V^a t^a + \frac{1}{2}(2g)^2 (D_\alpha V^b) V^c \underbrace{[t^b, t^c]}_{i f^{bca} t^a} \right\}$$

$$= \bar{D}^2 \left\{ D_\alpha V^a + \underbrace{ig f^{bca} (D_\alpha V^b) V^c} \right\} t^a$$

converts derivatives into gauge-covariant derivatives

$$= \left\{ -4i\lambda^a_\alpha + \left[ 4\delta^\beta_\alpha + 2i(\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta V^a_{\mu\nu} \right] \theta_\beta \right.$$

$$\left. - 4i\theta^2 \sigma^\mu_{\alpha\dot{\alpha}} \mathcal{D}_\mu \bar{\lambda}^{a\dot{\alpha}} \right\} t^a$$

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc} V^b_\mu V^c_\nu$$

$$\mathcal{D}_\mu \bar{\lambda}^{a\dot{\alpha}} = \partial_\mu \bar{\lambda}^{a\dot{\alpha}} - g f^{abc} V^b_\mu \bar{\lambda}^{c\dot{\alpha}}$$

(in the adjoint $(t^a)^{bc} = -if^{abc}$)

The field strength contribution to the Lagrangian is

$$\mathcal{L}_{gauge} = \frac{1}{64}\left[ W^{a\alpha} W^a_\alpha + W^{a\,+}_{\alpha} W^{a\alpha +}\right]_{F,\,WZ}$$

$$= -\frac{1}{4} V^a_{\mu\nu} V^{a\mu\nu} + i\lambda^a \sigma^\mu \mathcal{D}_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a$$

In the gauge - chiral superfield interaction Lagrangian the difference to earlier form is in the gauge covariant derivatives:

$$\mathcal{L}_{V-\Phi} = [\Phi^+ e^V \Phi]_D = (\mathcal{D}_\mu \varphi)^+ (\mathcal{D}^\mu \varphi) + i\psi \sigma^\mu \mathcal{D}^+_\mu \bar{\Psi} + F^+ F$$

$$+ i\sqrt{2} g (\varphi^+ t^a \lambda^a \psi - \bar{\Psi} t^a \bar{\lambda}^a \varphi) + g\varphi^+ t^a D^a \varphi$$

$$\mathcal{D}_\mu \varphi = \partial_\mu \varphi - ig\, t^a V^a_\mu \varphi$$

The superpotential part of the Lagrangian with $n$ chiral superfields $\Phi_i$ has to be invariant under the action of $G$:

$$\mathcal{L}_W = [W(\Phi_i) + h.c.]_F$$

The auxiliary fields are given by

$$F_i^+ = -\frac{\partial W}{\partial \varphi_i}$$

$$D^a = -\sum g \varphi_i^+ t^a_{ij} \varphi_j$$

and the scalar potential is

$$V = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 + \frac{1}{2} g^2 \sum_a \left| \sum_i \varphi_i^+ t^a_{ij} \varphi_j \right|^2$$

E.g. susy QCD with one quark flavour: take two chiral supermultiplets $\Phi_S, \Phi_T$, $\Phi_S$ in $\underset{\sim}{3}$ and $\Phi_T$ in $\underset{\sim}{\bar{3}}$

$$\Phi_S \to \Phi_S' = e^{-2ig_3 t^a \Lambda^a} \Phi_S$$

$$\Phi_T \to \Phi_T' = e^{2ig_3 t^{a*} \Lambda^a} \Phi_T\,, \quad \Phi_T^T \to \Phi_T^T e^{2ig_3 t^a \Lambda^a}$$

and $W = m \Phi_T^T \Phi_S$ is $SU(3)$ invariant

## Particle content of minimal supersymmetric extension of the SM

- in the simplest supersymmetric extension, the gauge and susy d.o.f. of the SM have their counterparts.
 In the Higgs-sector two doublets are needed instead of the one in the SM:

$$H_{1,Y=-1} = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} ; \quad H_{2,Y=1} = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$3 \to Z, W^\pm$
$h^0, H^0, A^0, H^\pm$

* Yukawa-terms come from superpotential $\Phi_1\Phi_2\Phi_3$ terms containing only left chiral fields (in SM: d-mass: $(\bar{u}\,\bar{d})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R$

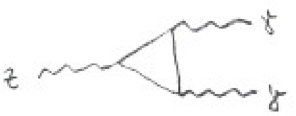
u-mass: $(\bar{u}\,\bar{d})_L \underbrace{\begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}}_{-i\tau_2\phi^*} u_R$ )

need $e_L, e_R$

$(e_R)^c = C\gamma^0\left[\frac{1}{2}(1+\gamma_5)e\right]^* = \frac{1}{2}(1-\gamma_5)\underbrace{C\gamma^0 e^*}_{e^c} = (e^c)_L$

$\psi^c = C\bar{\psi}^T$
$= C(\psi^+\gamma^0)^T$
$= C\gamma^0\psi^*$

* anomaly cancellation



$$\sum I_3^f Q_f^2 = 0$$

| SM | spartners W.I. eigenstate | | mass eigenstates | |
|---|---|---|---|---|
| $q = u, d, c, s, t, b$ | $\tilde{q}_L, \tilde{q}_R$ | squark | $\tilde{q}_1, \tilde{q}_2$ | |
| $l = e, \mu, \tau$ | $\tilde{l}_L, \tilde{l}_R$ | slepton | $\tilde{l}_1, \tilde{l}_2$ | |
| $\nu = \nu_e, \nu_\mu, \nu_\tau$ | $\tilde{\nu}$ | sneutrino | $\tilde{\nu}$ | |
| $g$ | $\tilde{g}$ | gluino | $\tilde{g}$ | |
| $W^\pm$ | $\tilde{W}^\pm$ | wino | $\tilde{\chi}^\pm_{1,2}$ | charginos |
| $H_1^+$ | $\tilde{H}_1^+$ | higgsinos | | |
| $H_2^-$ | $\tilde{H}_2^-$ | | | |
| $\gamma$ | $\tilde{\gamma}$ | fotino | $\tilde{\chi}^0_{1,2,3,4}$ | neutralinos |
| $Z$ | $\tilde{Z}$ | zino | | |
| $H_1^0$ | $\tilde{H}_1^0$ | higgsinos | | |
| $H_2^0$ | $\tilde{H}_2^0$ | | | |

($\tilde{g}, \tilde{W}^\pm, \tilde{Z}, \tilde{\gamma}$ = gauginos) (instead of $\gamma, Z \Rightarrow B, W^3$ often used $\Rightarrow$ partners $\tilde{B}, \tilde{W}$)

bino, neutral wino

## Supersymmetric version of the Standard Model

In the minimal susy extension of the SM one needs to introduce a chiral superfield for every chiral field of the SM or Higgs doublet. Two or more Higgs doublets are required for the higgsino anomalies to cancel. When the gauge symmetry is broken three of the scalar Higgses are absorbed by massive $W^\pm$, $Z$. The five physical Higgses are the neutral scalars $h^0$, $H^0$ and pseudoscalar $A^0$ and the charged $H^\pm$.

A vector superfield is needed for every gauge boson of the model. The kinetic terms and interactions of gauge supermultiplets with chiral matter and Higgs supermultiplets are ($Q = T_3 + Y/2$):

$$\mathcal{L}_{\substack{kin,\\ SU(2)\times U(1)}} = \Big[ Q^+ e^{ig\bar{\sigma}\cdot W + \frac{i}{3} g' B} Q + U^{c+} e^{-\frac{4}{3} ig' B} U^c + D^{c+} e^{\frac{2}{3} ig' B} D^c$$

$$+ L^+ e^{ig\sigma\cdot W - ig' B} L + E^{c+} e^{2ig' B} E^c + H_1^+ e^{ig\sigma\cdot W - ig' B} H_1$$

$$+ H_2^+ e^{ig\sigma\cdot W + ig' B} H_2 \Big]_D + \frac{1}{64}\Big[ W^{i\alpha} W^i_\alpha + W^{i+}_\alpha W^{i\alpha +} + 2B^\alpha B_\alpha \Big]_F$$

| | | $Q = I_3 + Y/2$<br>$Y$ |
|---|---|---|
| Chiral superfields : | $Q$ = quark doublet superfield | $+1/3$ |
| | $U^c$ = u-quark singlet " | $-4/3$ |
| | $D^c$ = d - " " " | $2/3$ |
| | $L$ = lepton doublet " | $-1$ |
| | $E^c$ = lepton singlet " | $2$ |
| | $H_1, H_2$ = Higgs doublet " | $H_{1\,Y=-1}$ , $H_{2\,Y=1}$ |

The $U(1)$ field strength $B_\alpha = \bar{D}^2 D_\alpha B$, $B$ = vector superfield

The $SU(2)$ " " $W^i_\alpha = \bar{D}^2 D_\alpha W^i + ig\epsilon^{ijk} \bar{D}^2 (D_\alpha W^j) W^k$

and correspondingly for $SU(3)$