Recall

Supergravity Lagrangian:

* nonrenormalizable

* $\mathcal{L} = \int d^4\theta \, K(\Phi^+ e^{2gV} \Phi) + \int d^2\theta \, (W(\Phi) + h.c.)$

$$+ \int d^2\theta \, (f_{ab}(\Phi) W^\alpha_a W_{\alpha b} + h.c.)$$

* φ^*, φ dependence in Kähler potential

$$G(\varphi^*, \varphi) = J(\varphi^*, \varphi) + \ln|W|^2$$

$$J(\varphi^*, \varphi) = -3 \ln(-K/3)$$

[E.g. choose $K = -3\exp(-\Phi_i \Phi^{i+}/3)$

$\Rightarrow G = \varphi_i \varphi^{i*} + \ln|W|^2$]

* $\langle \delta\psi_i \rangle = -\sqrt{2}\, e^{G/2} (G^{-1})_i{}^j G_j \xi \neq 0$

$\Rightarrow$ generalization of F-term breaking

$\langle \delta\lambda_a \rangle = \frac{i}{2} g G^i (T_a)_{ij} \varphi_j \xi \neq 0$

$\Rightarrow$ generalization of D-term breaking

* scalar potential

$$V = -e^G(3 - G_i (G^{-1})^i{}_j G^j) + \frac{g^2}{2} (\mathrm{Re} f^{-1}_{ab}) G^i (T_a)_{ij} \varphi_j G^k (T_a)_{kl} \varphi_l$$

* $G = \varphi^{i*}\varphi_i + \ln|W|^2$; $f_{ab}(\varphi) = \delta_{ab}$

$\Rightarrow G^i = \varphi^{i*} + \frac{1}{W} \frac{\partial W}{\partial \varphi_i}$; $G^i{}_j = \delta^i{}_j$

* Super-Higgs phenomenon

$$\mathcal{L}_{F.M} = \frac{i}{2} e^{G/2} \bar{\Psi}'_\mu \sigma^{\mu\nu} \psi'_\nu - \frac{1}{2} e^{G/2} (G^{ij} + \frac{1}{3} G^i G^j) \bar{\Psi}_i \Psi_j$$

Susy breaking in hidden sector

Hidden sector = part of the theory which couples to observable particles only by gravitational interactions

| Note: ϕ does not have gauge interactions

As an example, consider the Polonyi superpotential (1977)

$$W(\Phi) = m^2(\Phi + \beta), \quad m, \beta \text{ real}$$

and

$$G = \varphi^*\varphi + \ln |W|^2$$

Then

$$V = e^{\varphi^*\varphi}\left(\left| \frac{\partial W}{\partial \varphi} + \varphi^* W \right|^2 - 3|W|^2 \right)$$

$$= e^{\varphi^*\varphi} m^4 \left[|1 + \varphi^*\varphi + \beta\varphi^*|^2 - 3|\varphi + \beta|^2 \right]$$

Choose then $\beta = 2 - \sqrt{3}$ in which case

$$V_{minimum} = V(\langle\varphi\rangle = \sqrt{3} - 1)$$

$$= e^{(\sqrt{3}-1)^2} m^4 \left\{ (1 + 3 + 1 - 2\sqrt{3} + 2\sqrt{3} - 2 - 3 + \sqrt{3})^2 - 3 \right\}$$

$$= 0,$$

$$W G^i = \frac{\partial W}{\partial \varphi} + \varphi^* W = \sqrt{3}\, m^2 \neq 0 \quad \text{so that susy is broken}$$

The gravitino mass is

$$m_{3/2} = e^{G_0/2} m_p = e^{\frac{1}{2}\left[\langle\varphi\rangle^2 + \ln \left|\frac{m^2}{m_p^2}(\langle\varphi\rangle + \beta)\right|^2\right]} m_p$$

$$= e^{(\sqrt{3}-1)^2/2} \frac{m^2}{m_p^2} m_p$$

The susy breaking scale, defined by

$$\langle \delta\phi_i \rangle = \left(- e^{G/2} (G^{-1})^j_i\, G_j\, \xi \right)\sqrt{2}$$

$$= \left(- M_S^2\, \xi \right)\sqrt{2} \quad \text{is} \quad M_S^2 = e^{G/2}\left(\varphi^* + \frac{1}{W}\frac{\partial W}{\partial \varphi} \right)$$

$$= m_{3/2}\sqrt{3}\, m_p$$

(true for models with $V = 0$)

The gravitino mass $m_{3/2} = \frac{M_S^2}{\sqrt{3}\, m_{Planck}} \sim 100$ GeV implies the susy breaking scale $M_S \sim 10^{10}$ GeV.

The mass terms for the fermions

$$|\det e|^{-1} \mathcal{L}_F^m = \tfrac{i}{2} m_{3/2} \bar{\Psi}'_\mu \sigma^{\mu\nu} \Psi'_\nu$$

$G^{ij} + \frac{1}{3} G^i G^j = -1 + \frac{1}{3}\sqrt{3}\sqrt{3} = 0$
(goldstino is eaten by gravitino)

The mass terms for scalars from V ($\varphi = \varphi_0 + \varphi'$, $\varphi_0 =$ vev of φ)

$$\mathcal{L}_B^m = -2 m_{3/2}^2 \varphi'^* \varphi' - 2(\sqrt{3}-1) m_{3/2}^2 (\varphi'\varphi' + \varphi'^*\varphi'^*)$$

write $\varphi' = \frac{1}{\sqrt{2}}(A + iB)$

$$m_A^2 = 2\sqrt{3}\, m_{3/2}^2$$

$$m_B^2 = 2(2-\sqrt{3})\, m_{3/2}^2$$

Here

$$\text{str } M^2 = \sum (-1)^{2J}(2J+1) m_J^2 = -4 m_{3/2}^2 + m_A^2 + m_B^2 = 0$$

Generally

$\text{Str } M^2 = 2(N-1) m_{3/2}^2$, $N =$ # chiral superfields

requiring scalars to be on the average heavier than fermions

(If spin $(\frac{1}{2}, 1)$ multiplets are included,

$$\text{Str } M^2 = (N-1)\left[2m_{3/2}^2 - \frac{D^a D^a}{m^2}\right] - 2\tilde{g}_\alpha D^\alpha \, \text{Tr} T^\alpha \;)$$

Note that splitting in multiplets is characterized by $m_{3/2}$, <u>not</u> M_S.

Low energy effects of susy breaking

Let us denote by $Z_{i,j,..}$ the hidden sector superfields (with scalar $z_{i,j,k,\dots}$) and by Y_r the observable sector superfields (with scalar $y_{r,s,..}$). The superpotential is assumed to be additive:

$$W(Z_i, Y_r) = \overline{W}(Z_i) + \widetilde{W}(Y_r)$$

(Arnowitt, Chamseddine, Nath; Barbieri, Ferrara, Savoy 1982)

E.g. assume Z_i inv. in $SU(3)\times SU(2)\times U(1) \subset G_0$, Y_r inv in G_H = hidden sector gauge group

and Kähler potential

$$G = \frac{1}{m_P^2}\left(z^{i*}z_i + y^{r*}y_r\right) + \ln\left(|W|^2/m_P^6\right)$$

The tree level effective potential is then

$$V = e^{(z^{i*}z_i + y^{r*}y_r)/m_P^2}\left\{\left|\frac{\partial\overline{W}}{\partial z_i} + \frac{z^{i*}}{m_P^2}(\overline{W}+\widetilde{W})\right|^2 + \left|\frac{\partial\widetilde{W}}{\partial y_r} + \frac{y^{r*}}{m_P^2}(\overline{W}+\widetilde{W})\right|^2 - \frac{3}{m_P^2}\left|\overline{W}+\widetilde{W}\right|^2\right\}$$

At low energy, replace z_i by $\langle z_i\rangle|_{\text{minimum}}$ and take leading order in $\frac{1}{m_P}$, with

$$\langle z_i\rangle = a_i m_P$$
$$\langle\overline{W}\rangle = \mu m_P^2$$
$$\left\langle\frac{\partial\overline{W}}{\partial z_i}\right\rangle = c_i \mu m_P$$

Then the gravitino mass

$$m_{3/2} \cong e^{G_0/2} m_P \quad ; \quad \frac{G_0}{2} = \frac{|a_i|^2 m_P^2}{2m_P^2} + \frac{1}{2}\left(\ln\frac{\mu^2}{m_P^2}\right) = \frac{|a_i|^2}{2} + \ln\frac{\mu}{m_P}$$

$$= e^{|a_i|^2/2}\mu$$

In the low energy limit ($\frac{1}{m_P}$ neglected, $m_P \to \infty$ with $m_{3/2}$ fixed)

$$V = e^{|a_i|^2} \left\{ \left| \frac{\partial \tilde{W}}{\partial y_r} \right|^2 + \mu^2 |y_r|^2 + \mu \left(y_r \frac{\partial \tilde{W}}{\partial y_r} + (A-3)\tilde{W} + cc \right) \right\}$$

where $A = (c_i^* + a_i) a_i^*$

$\hat{W} \equiv e^{|a_i|^2/2} \tilde{W}$

$$= \underbrace{\left| \frac{\partial \hat{W}}{\partial y_r} \right|^2}_{\text{usual } V \text{ with superpot. } \hat{W}} + \underbrace{m_{3/2}^2 |y_r|^2 + m_{3/2} \left(y_r \frac{\partial \hat{W}}{\partial y_r} + (A-3)\hat{W} + c.c. \right)}_{\text{susy breaking terms}}$$

One has extra mass term for y_r resulting in mass splitting $\propto m_{3/2}$ between bosons and fermions of a chiral susy multiplet.

For Polonyi superpotential ($c_i = 1$, $a_i = \sqrt{3} - 1$)

$$A = 3 - \sqrt{3}$$

Note: for $V=0$ at the minimum
$|c_i \mu m_P + a_i^* \mu m_P|^2 = 3\mu^2 m_P^2$
$\Rightarrow \sum_i |c_i + a_i^*|^2 = 3$

Note that A and $m_{3/2}$ are defined at the energy scale where supergravity is valid. They must be run to low energies to enable comparison with measurements.

E.g. take $\hat{W}$ trilinear in chiral superfields Y_r (e.g. $\hat{W} = XYZ$; trilinear $\hat{W}$ phenomenologically attractive, since it does not contain arbitrary mass parameters)
Then

$$V = \left| \frac{\partial \hat{W}}{\partial y_r} \right|^2 + m_{3/2}^2 |y_r|^2 + A m_{3/2} (\hat{W} + \hat{W}^*)$$

In V one has a universal susy breaking mass $m_{3/2}$, which is useful for FCNC's. This assumption is often used, but models exist with nonuniversal scalar masses. (Universal masses arise when observable sector kinetic terms are minimal.)

In general, also D-terms are present:

$$V = \left| \frac{\partial \hat{W}}{\partial y_r} \right|^2 + m_{3/2}^2 |y_r|^2 + m_{3/2} \left(y_r \frac{\partial \hat{W}}{\partial y_r} + (A-3)\hat{W} + cc \right) + \frac{1}{2} \mathrm{Re} f_{ab}^{-1} D_a D_b$$

$$D_a = g y^{r*} (T_a)_{rs} y_s$$