Supersymmetry, autumn 2010, exercise 12
(please return on Dec. 9)

1. Explain the origin of the different terms in the chargino Lagrangian:

$$L_{\chi^{\pm}} = ig(\lambda^{+}\tilde{H}_1^{-}v_1 + \lambda^{-}\tilde{H}_2^{+}v_2) + M_2\lambda^{+}\lambda^{-} - \mu\tilde{H}_2^{+}\tilde{H}_1^{-} + h.c.,$$

   where $\lambda^{\pm} = (\lambda_1 \mp i\lambda_2)/\sqrt{2}$. Find the chargino mass matrix

$$X = \begin{pmatrix} M_2 & m_W\sqrt{2}\sin\beta \\ m_W\sqrt{2}\cos\beta & \mu \end{pmatrix}.$$

   starting from the Lagrangian, and using the Dirac spinors

$$\Psi^{+} = \begin{pmatrix} -i\lambda^{+} \\ \tilde{H}_2^{+} \end{pmatrix}, \quad \Psi^{-} = \begin{pmatrix} -i\lambda^{-} \\ \tilde{H}_1^{-} \end{pmatrix}.$$

   Hint: Write the Lagrangian in the form

$$L_{\chi^{\pm}} = -\frac{1}{2}(\Psi^{+T}\Psi^{-T})\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \Psi^{+} \\ \Psi^{-} \end{pmatrix}.$$

2. Find the chargino masses using the chargino mass matrix.
   Hint. Use $\chi_i^{+} = V_{ij}\Psi_j^{+}$ and $\chi_i^{-} = U_{ij}\Psi_j^{-}$ $(i,j = 1,2)$, where $U^{*}XV^{-1} = M_D$ =diagonal. Note that $M_D^2 = VX^{+}XV^{-1} = U^{*}XX^{+}(U^{*})^{-1}$.

3. Find the locally symmetric action for nonabelian gauge theory by Noether procedure starting from the globally symmetric action

$$S_0 = -\frac{1}{4}\int G_a^{\mu\nu}G_{\mu\nu}^{a}d^4x,$$

   where $G_a^{\mu\nu} \equiv \partial^{\mu}A_a^{\nu} - \partial^{\nu}A_a^{\mu}$.
   Hint. See lectures or Bailin & Love.