Due to Friday 17 February 12.00.

1. Evaluate two correction terms of an ideal Fermi-Dirac gas to the density correlation function h of the Maxwell-Boltzmann gas (for which $h = 0$).

2. Evaluate the pair correlation function of the photon gas.

3. Let the probability density of the density fluctuations in the wave-vector space be

$$P[\delta n] \;\propto\; \mathrm{e}^{-\beta\tilde{F}} \;\propto\; \exp\left[-\frac{1}{2TV}\sum_{\boldsymbol{q}}{}' \left(f_1 + f_2\boldsymbol{q}^2\right)|\delta n(\boldsymbol{q})|^2\right], \tag{1}$$

where

$$\delta n(\boldsymbol{q}) = \int d\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}}\delta n(\boldsymbol{x})\,.$$

The prime in the wave-vector sum means that the term with $\boldsymbol{q} = 0$ is omitted. Note that due to $\delta n^*(\boldsymbol{q}) = \delta n(-\boldsymbol{q})$ each term in the sum is actually calculated twice.

Calculate the Fourier transform of the density correlation function

$$C_{nn}(\boldsymbol{q},\boldsymbol{q}') = \int d\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \int d\boldsymbol{x}'\, e^{-i\boldsymbol{q}'\cdot\boldsymbol{x}}\langle\delta n(\boldsymbol{x})\delta n(\boldsymbol{x}')\rangle$$

in this system.

4. Calculate the static response function of density $n = \langle\hat{\psi}^+(\boldsymbol{x})\hat{\psi}(\boldsymbol{x})\rangle$ to the perturbation $\Delta H = a\hat{\psi}^+(\boldsymbol{x})\hat{\psi}(\boldsymbol{x})$ in an ideal quantum gas.

5. Calculate the Fourier transform $\chi_{nn}(k,\omega)$ of the dynamic density-density response function of one-dimensional ideal electron gas at zero temperature. Note that in one dimension electrons are spinless fermions. Here, it is safest to use the commutator representation

$$\chi_{nn}(x - x', t - t') = \frac{i}{\hbar}\theta(t - t')\big\langle\big[\hat{n}(x,t),\hat{n}(x',t')\big]\big\rangle\,.$$

To change the order of integration a cutoff factor $e^{-\varepsilon t}$, $\varepsilon \to 0+$ is needed in the Fourier transform, wherefrom $\varepsilon \to 0$ remains in the result.