

1. The differential cross section for pion production  $pp \rightarrow d\pi^+$  at certain energy is

$$d\sigma/d\Omega = a + b \cos^2 \theta,$$

where  $a = 0.05$  mb and  $b = 0.25$  mb.

- (a) What is the total cross section?
- (b) A detector is placed 50 cm away from the target at  $90^\circ$  angle. The effective area of the detector is  $1 \text{ cm}^2$ . The target is  $10^{-4} \text{ cm}$  thick and it contains  $10^{22} \text{ protons/cm}^3$ . At what rate does the detector count pions, if the incident proton beam is  $1 \mu\text{A}$ ? (You don't have to take into account the difference between center of mass and laboratory frames.)
2. A rough estimate for the validity of the Born approximation in the case of a spherically symmetric potential  $V = V(r)$  can be obtained by comparing the two terms on the right hand side of the integral equation

$$\Psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} U(r') \Psi_{\mathbf{k}}^{(+)}(\mathbf{r}')$$

with each other at point  $\mathbf{r} = 0$ . In the spirit of the Born approximation, replace  $\Psi_{\mathbf{k}}^{(+)}(\mathbf{r}')$  in the second term by a plane wave  $e^{i\mathbf{k}\cdot\mathbf{r}'}/(2\pi)^{3/2}$ . Show that requiring the spherical wave term to be much smaller than the plane wave term leads to the condition

$$\frac{2\mu}{\hbar^2} \frac{1}{k} \left| \int_0^\infty dr V(r) e^{ikr} \sin kr \right| \ll 1.$$

Qualitatively, for what kind of potentials and energies, according to this condition, is the Born approximation valid?

3. Compute the scattering amplitude in the first Born approximation for the following potentials.
- (a)  $V(r) = V_0/(r^2 + a^2)$
- (b)  $V(r) = -V_0 \delta(r - a)$

Is the total cross section finite?

This is the last problem set. Next week (50, starting 12.12.) there will be no lectures, only exercise sessions. On Thursday 15 Dec only the latter session (10–12) will be held, there is *no exercise session on Thursday at 8–10*. Friday's session will be held as usual.

Please give course feedback either in WebOodi or in person, it will be much appreciated.