Hydrodynamics, spring 2010

Exercise 9. Thu, 8.4.2010 at 14–16 in Exactum, C220.

This time the exercises should **not be submitted** before the session. Points will be given based on a list circulated in the beginning of the session, where everyone marks those problems that he/she has calculated and is willing to present on the blackboard. Three points for each problem plus one for the one actually presenting it, if the solution is correct (or "almost correct").

1. Consider the perturbation of a two-fluid interface including the effect of surface tension. The effect of surface tension is to make the pressure at a point infinitesimally below the surface higher than the pressure at a point infinitesimally above the surface by an amount $-T(\partial^2\xi_1/\partial x^2)$, where T is the surface tension. Show that this leads to the dispersion relation

$$\frac{\omega}{k} = \frac{\rho U + \rho' U'}{\rho + \rho'} \pm \left[\frac{g}{k}\left(\frac{\rho - \rho'}{\rho + \rho'} + \frac{k^2 T}{g(\rho + \rho')}\right) - \frac{\rho\rho'(U - U')}{(\rho + \rho')^2}\right]^{1/2}$$

   Explain what $\partial^2\xi_1/\partial x^2$ stands for, geometrically (for small ξ_1).

2. Apply the dispersion relation of problem 1 to the following problems:

   (a) What is the phase velocity of surface waves on the air–water interface (NTP)? Show that this velocity attains a minimum value at a particular wavelength. Given that $T = 74\,\mathrm{mN\,m^{-1}}$ (milli-Newtons per meter), calculate this wavelength.

   (b) Consider the Kelvin–Helmholtz instability on the air-water interface (NTP). What is the critical value of $|U - U'|$ above which the instability starts in the sense of some wavenumber becoming unstable.

3. Consider surface waves at the air-water interface, with the water having a finite depth h. Assuming that the normal component of velocity becomes zero at the bottom, and neglecting surface tension, show that

$$\omega^2 = gk \tanh kh.$$

   If the water is shallow (i.e., $h \ll k^{-1}$) show that the propagation velocity of the surface wave is $\propto \sqrt{h}$. Use this result to explain the formation of surf on the seashore.

4. The convective granules near the solar surface have the typical size of about 10^3 km and typical velocity of about 1 $\mathrm{km\,s^{-1}}$. Make an estimate of the coefficient of turbulent diffusion. Sunspots are structures on the solar surface with sizes of about 10^4 km. If sunpots decay due to turbulent diffusion, make an estimate of the lifetime of a sunspot.

These are the last exercises of the course.