COSMOLOGY II Homework 12 Fall 2011

Due on Monday December 12 by 14.15.

1. **Cosmic variance.** Assume that the CMB multipole coefficients a_{lm}, $m = -l, \ldots, l$, are independent Gaussian random variables for $m = 0, \ldots, l$, with variance $\langle |a_{lm}|^2 \rangle = C_l$ (same for all m). The coefficient a_{l0} is real but the other a_{lm} are complex, and $a_{l,-m} = a^*_{lm}$. The observed angular power spectrum $\hat{C}_l$ is defined as

$$\hat{C}_l \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2 .$$

Clearly, $\langle \hat{C}_l \rangle = C_l$. Calculate the cosmic variance, defined as the expectation value

$$\langle (\hat{C}_l - C_l)^2 \rangle .$$

(Exercise 7.2 may be helpful.)

2. **Positions of the acoustic peaks.**

a) Calculate the sound horizon at decoupling,

$$r_s = (1+z)^{-1} \int_0^{t_{\mathrm{dec}}} dt \frac{c_s}{a} \ ,$$

in terms of z_{dec}, H_0, h, $\omega_m \equiv \Omega_{m0} h^2$ and $\omega_b \equiv \Omega_{b0} h^2$. Assume a constant speed of sound, $c_s = c_s(t_{\mathrm{dec}})$, but include the effect of radiation and matter components in the expansion law. Neglect neutrino masses.

b) What is the separation ℓ_A between the acoustic peaks in the CMB angular power spectrum C_ℓ for the cases $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1 - \Omega_m$?

c) Give the numerical values for of the previous quantities for $z_{\mathrm{dec}} = 1090$, $h = 0.7$, $\Omega_{m0} = 0.3$ and $\omega_b = 0.02$. Give also the numerical value of the comoving sound horizon.

3. **The effect of a varying sound speed.** Same as the previous problem, but now include the effect of the varying speed of sound.